CURVE OF THE REESTABLISHMENT OF THE PRESSURE IN A BOREHOLE IN A FRACTURED POROUS RESERVOIR

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It is shown that the slope of the initial segment of the curve for reestablishment of the pressure cannot be less than half of the asymptotic slope over a long period of time; the article gives the limits of the errors in determination of the characteristic size of the block from the known lag time for reestablishment of the pressure.

In the article of Warren and Root [1], based on the differential equations for the filtration of a liquid in a fractured porous medium

$$\frac{k_1}{\mu}\nabla^2 p_1 = \beta_1{}^* \frac{\partial p_1}{\partial t} + \beta_2{}^* \frac{\partial p_2}{\partial t}, \quad \beta_2{}^* \frac{\partial p_2}{\partial t} = \frac{\alpha}{\mu}(p_1 - p_2) \tag{0.1}$$

first proposed by G. I. Barenblatt and Yu. P. Zheltov [2], a dependence is obtained, describing the reestablishment of the pressure in a borehole in an infinite stratum in the form

$$P_b = \frac{1}{2}\left\{\ln\tau + 0.80908 + E_i\left[-\frac{\lambda\tau}{\omega(1-\omega)}\right] - E_i\left(-\frac{\lambda\tau}{1-\omega}\right)\right\} \tag{0.2}$$

Here

$$P_b = \frac{2\pi k_1 h}{q_0 \mu}(p_1 - p_{10}), \quad \tau = \frac{k_1 t}{\mu(\beta_1{}^* + \beta_2{}^*) r_b{}^2}, \quad \lambda = \frac{r_b^2}{\eta}, \quad \omega = \frac{\beta_1{}^*}{\beta_1{}^* + \beta_2{}^*}$$

k_1 is the permeability coefficient of the fractured medium; μ is the dynamic viscosity of the liquid; ∇^2 is a Laplace operator; $\beta_1{}^*$ and $\beta_2{}^*$ are the elastic capacities of the fractured medium and of the porous block, respectively; p_1 and p_2 are the pressures at a distance r from the axis of the borehole, and in the blocks, respectively; α is a dimensionless coefficient, characterizing the fractured porous medium; P_b is the increase in the pressure in the borehole, dimensionless; q_0 is the fully established output of a borehole up to its shutdown; h is the thickness of the stratum; p_{10} and p_{1b} are the stope pressures before and after shutdown of the borehole; t is the time, calculated from the moment of shutdown of the borehole; r_b is the radius of the borehole; τ is the dimensionless time; η is the characteristic parameter of the fractured porous medium; λ and ω are dimensionless parameters [1].

For $\omega \to 0$ it follows from (0.2) that*

$$P_b = {}^1/_2\,[\ln\tau + 0.80908 - E_i(-\lambda\tau)] \tag{0.3}$$

Dependence (0.2) and its limiting case, dependence (0.3), are illustrated graphically in Fig. 1 (except for the dotted line); here $\lambda = 5 \cdot 10^{-6}$ (according to the data of [1]). A characteristic feature of the dependence (0.2) is the presence, starting at a certain value $\omega \approx 0.002$, of a horizontal segment on the transitional line, connecting the straight lines with parallel slopes. The length of this segment increases with an increase in ω. Therefore, for example, R. I. Medvedskii [4], on the basis of an analysis of dependence (0.3)

*E. A. Avakyan [3] and R. I. Medvedskii [4] arrived earlier at this type of dependence.

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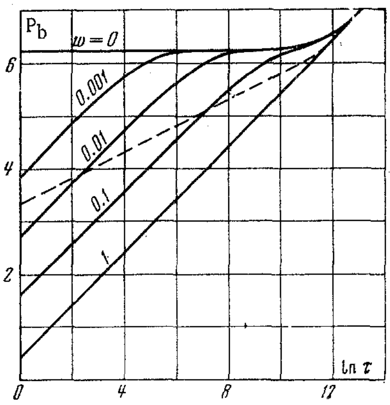

Fig. 1

in more explicit form, drew a conclusion with respect to the transient stabilization of the initial segment of the actual curve for the reestablishment of the pressure of a fractured porous stratum. However, it is evident that such a form of the initial section is a result of the adopted form of the flow function. In the derivation of the starting differential equations (0.1) for (0.2) and (0.3), it was assumed that the value of the return flow of liquid is proportional to the difference in pressure in the two media: the fractured and the porous.* Therefore, it is of interest to estimate to what degree this assumption corresponds to the actual situation with respect to reestablishment of the pressure.

It is obvious that the return-flow function is determined by the geometric form of the block and by the ratio of the surface of the block to its volume. It is well known [6] that the greater the ratio of the surface of a body to its volume, other conditions being equal, the more rapidly it is cooled or heated. It must be postulated that analogous phenomena, but with respect to the pressure, will exist also for porous bodies, saturated by a liquid. In comparison with bodies of other form (with equal characteristic linear dimensions), a sphere has the greatest ratio of surface to volume, while an infinite plate has the least. Consequently, strata in which the return-flow functions correspond to these forms of solids will have pressure reestablishment curves with initial segments whose positions will be limiting for strata with blocks of any given form.

As confirmation of what has been said above, let us consider two problems involving the reestablishment of the pressure in a borehole. In the first problem we shall assume that the return-flow function corresponds to unbounded porous plates, while in the second problem we shall assume that the return flow of the liquid into the blocks takes place as if they had the form of a sphere.

1. Let us consider the first problem. In general form, the equation for the not fully established axisymmetric filtration of a liquid in a fractured porous medium is

$$\frac{k_1}{\mu}\left(\frac{\partial^2 p_1}{\partial r^2}+\frac{1}{r}\frac{\partial p_1}{\partial r}\right)=\beta_1{}^*\frac{\partial p_1}{\partial t}+q$$

where q is the value of the return flow of the liquid from the cracks into the blocks.

Substituting the value of the return flow $q = \beta_2{}^* \partial p_2{}^\circ/\partial t$, where $p_2{}^\circ$ is the mean pressure in the plate along the z axis at a distance from the axis of the borehole, we obtain,

$$\frac{k_1}{\mu}\left(\frac{\partial^2 p_1}{\partial r^2}+\frac{1}{r}\frac{\partial p_1}{\partial r}\right)-\beta_1{}^*\frac{\partial p_1}{\partial t}-\beta_2{}^*\frac{\partial p_2{}^\circ}{\partial t}=0 \tag{1.1}$$

The z axis has its origin in the middle of the plate and is perpendicular to its surface.

Assuming that, up to its shutdown, the borehole operated under steady-state conditions with an output q_0, we express the pressures p_1 and $p_2{}^\circ$ in terms of

$$u_1(r,t)=p_1(r,t)-p_0(r),\quad u_2{}^\circ(r,t)=p_2{}^\circ(r,t)-p_0(r)$$

where $p_0(r)$ is the steady-state pressure distribution before the shutdown of the borehole. Then, instead of (1.1), we have

$$\frac{\partial^2 u_1}{\partial r}+\frac{1}{r}\frac{\partial u_1}{\partial r}-\frac{1}{\varkappa}\left(\frac{\beta_1{}^*}{\beta_2{}^*}\frac{\partial u_1}{\partial t}+\frac{\partial u_2{}^\circ}{\partial t}\right),\quad \varkappa=\frac{k_1}{\mu\beta_2{}^*} \tag{1.2}$$

To restore the pressure in a borehole with negligible radial filtration of the liquid in the strata, the initial and boundary conditions will be

$$\begin{gathered}u_1(r,t)\big|_{r=0}=0,\quad u_2{}^\circ(r,t)\big|_{t=0}=0\\ u_1(r,t)\big|_{r\to\infty}=0,\quad r\frac{\partial u_1(r,t)}{\partial r}\bigg|_{r=r_b}=-\frac{q_0\mu}{2\pi k_1 h}\end{gathered} \tag{1.3}$$

* The difficulty in evaluating this assumption is pointed out in the article of I. A. Volkov [5].

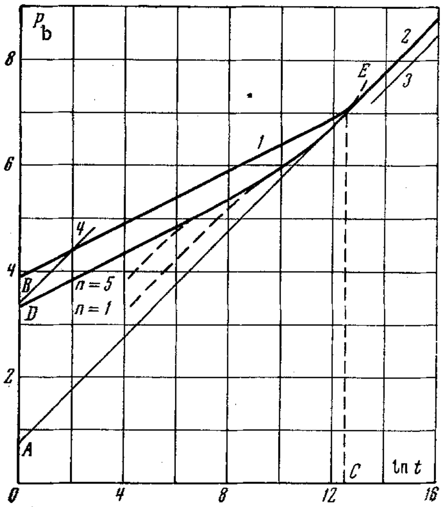

Fig. 2

Applying a Laplace transform with respect to the variable t to (1.2) and (1.3), we obtain

$$U_1'' + \frac{1}{r} U_1' - \frac{s}{\varkappa}\left(\frac{\beta_1^*}{\beta_2^*} U_1 + U_2^\circ\right) = 0 \tag{1.4}$$

$$U_1(r, s)\big|_{r\to\infty} = 0, \quad rU_1'(r, s)\big|_{r=r_b} = -\frac{1}{s}\frac{q_0\mu}{2\pi k_1 h} \tag{1.5}$$

Let us find U_2°. To this end, we solve the one-dimensional equation of the piezoconductivity for the return flow of a liquid into an unbounded porous medium (a stratum). In this case we shall assume that the velocity of the filtration of the liquid into the plate is perpendicular to its surface.

Thus, we have

$$\frac{\partial^2 u_2}{\partial z^2} - \frac{1}{\varkappa_2}\frac{\partial u_2}{\partial t} = 0, \quad \varkappa_2 = \frac{k_2}{\mu\beta_2^*} \tag{1.6}$$

with the following initial and boundary conditions:

$$u_2(z, r, t)\big|_{t=0} = 0$$
$$u_2(z, r, t)\big|_{z=R} = u_1(r, t), \quad \frac{\partial u_2(z, r, t)}{\partial z}\bigg|_{z=0} = 0 \tag{1.7}$$

Here R is the half-thickness of the plate, i.e., the characteristic linear dimension; k_2 is the permeability coefficient of the porous medium.

Applying a Laplace transform with respect to the variable t to (1.6) and (1.7), we find [7]

$$U_2 = U_1 \operatorname{ch}\sqrt{\frac{s}{\varkappa_2}}\, z \,/\operatorname{ch}\delta, \quad \delta = R\sqrt{\frac{s}{\varkappa_2}}$$

Hence

$$U_2^\circ = \frac{1}{R}\int_0^R U_2\, dz = U_1 \operatorname{th}\delta/\delta \tag{1.8}$$

Taking account of (1.8). Eq. (1.4) is written as

$$U_1'' + \frac{1}{r}U_1' - \frac{s}{\varkappa}\left(\frac{\beta_1^*}{\beta_2^*} + \frac{\operatorname{th}\delta}{\delta}\right)U_1 = 0 \tag{1.9}$$

Obviously, the solution of (1.9) which satisfies the boundary conditions (1.5) is

$$U_1 = \frac{q_0\mu}{2\pi k_1 h}\frac{K_0(\sqrt{\xi(s)}\,r)}{s\sqrt{\xi(s)}\,r_b K_1(\sqrt{\xi(s)}\,r_b)}, \quad \xi(s) = \frac{s}{\varkappa}\left(\frac{\beta_1^*}{\beta_2^*} + \frac{\operatorname{th}\delta}{\delta}\right) \tag{1.10}$$

Approximately (for $\sqrt{\xi(s)}rc < 0.01$), replacing the modified Bessel functions of the first kind, of zero and first orders, by their asymptotic expressions for a small argument, for the borehole we obtain ($r = r_b$)

$$U_1(r_b, s) = \frac{q_0\mu}{2\pi k_1 h}\frac{1}{s}\left(-\ln\frac{\gamma r_b}{2}\sqrt{\xi(s)}\right) \quad (\gamma = 1.781\ldots)$$

Let us consider the case when the value of β_1^* may be neglected. Then, after simple transformations

$$U_1(r_b, s) = \frac{q_0\mu}{2\pi k_1 h}\left\{\frac{1}{s}\ln\frac{4\varkappa R}{\gamma^2\sqrt{\varkappa_2}\,r_b^2} - \frac{1}{2}\frac{\ln s}{s} + 2\sum_{n=1}^{\infty}\frac{1}{2n-1}\frac{\exp[-2(2n-1)\delta]}{s}\right\}$$

Going over to the inverse transform, and introducing the pressure P_b, we obtain

$$P_b = \frac{1}{2}\left[\ln\frac{4\varkappa t}{\gamma r_b^2} - \ln\sqrt{\gamma F_0} + 2\sum_{n=1}^{\infty}\frac{1}{2n-1}\operatorname{erfc}\frac{2n-1}{\sqrt{F_0}}\right] \quad (F_0 = \varkappa_2 t/R^2) \tag{1.11}$$

Figure 2 shows a curve of the dependence (1.11) (solid line). This curve was plotted from the following starting data: R = 1 m; r_b = 0.1 m; $\varkappa = 2 \cdot 10^{-2}$ m^2/sec; $\varkappa_2 = 2 \cdot 10^{-6}$ m^2/sec.

On the plotted curve, there are two separate segments, 1 and 2. Segment 1 is described by the dependence

$$P_b = \frac{1}{2}\left(\ln \frac{4\varkappa t}{\gamma r_b^2} - \ln \sqrt{\gamma F_0}\right) \tag{1.12}$$

since, at sufficiently small values of F_0, the sum

$$2 \sum_{n=1}^{\infty} \frac{1}{2n-1} \operatorname{erfc} \frac{2n-1}{\sqrt{F_0}} \tag{1.13}$$

by virtue of the properties of the function erfc ξ, for small values of the argument ξ, approaches zero.

Segment 2 is described by the dependence

$$P_b = \frac{1}{2} \ln \frac{4\varkappa t}{\gamma r_b^2} \tag{1.14}$$

since, starting from sufficiently large values of F_0, the sum (1.3) will no longer be equal to zero and, as calculations show, rapidly approaches $\ln \sqrt{\gamma F_0}$.

As can be seen from (1.12) and (1.14), segment 1 has a slope which is twice as small as the slope of segment 2, and corresponds to a situation in the stratum when the blocks (plates) already behave as semi-infinite bodies. The intercepts on the axis of ordinates are

$$OA = \frac{1}{2} \ln \frac{4\varkappa}{\gamma r_b^2}, \quad AB = \frac{1}{4} \ln \frac{R}{\sqrt{\gamma \varkappa_2}}$$

The abscissa of the point of intersection of the prolongations of segments 1 and 2 is equal to 4AB. The lag time for the reestablishment of the pressure, τ_3, determined using the method proposed in [4], for the model of a stratum under consideration, assumes the following meaning: $\tau_3 = R^2/\varkappa_2$. Taking account of this equality, the dependence (1.11) is shown by the dotted line on Fig. 1.

We note that, instead of (1.11), the following expression can be used for P_b:

$$P_b = \frac{1}{2}\left\{\ln \frac{4\varkappa t}{\gamma r_b^2} + \sum_{n=1}^{\infty} E_i(-n^2\pi^2 F_0) - \sum_{n=1}^{\infty} E_i\left[-\left(\frac{2n-1}{2}\right)^2 \pi^2 F_0\right]\right\}$$

if th δ is represented in the form of the ratio of the infinite products [8].

For small values of the complex δ, which corresponds to large values of F_0, Eq. (1.9) is written as follows:

$$U_1'' + \frac{1}{r} U' - \frac{s}{\varkappa}\left(\frac{\beta_1^*}{\beta_2^*} + 1\right) U_1 = 0$$

and its solution will be

$$P_b = \frac{1}{2} \ln \frac{4k_1 t}{\gamma \mu (\beta_1^* + \beta_2^*) r_b^2} \tag{1.15}$$

Graphically, it is illustrated by the straight line 3 (Fig. 2), passing somewhat below line 2, and coinciding with it at $\beta_1^* \to 0$.

For large values of δ, when th δ/δ becomes negligibly small in comparison with β_1^*/β_2^*, from (1.9) we obtain

$$U_1'' + \frac{1}{r} U_1' - \frac{s}{\varkappa_1} U_1 = 0, \quad \varkappa_1 = \frac{k_1}{\mu \beta_1^*}$$

and the solution will be

$$P_b = \frac{1}{2} \ln \frac{4\varkappa_1 t}{\gamma 4_b^2} \tag{1.16}$$

On Fig. 2, this solution is shown by line 4. The distance along the ordinate between lines 4 and 3 is determined by the difference between (1.16) and (1.15) $\Delta P_b = -\ln\sqrt{\omega}$, i.e., ω is determined by the same expression as in [1]. This argues that, at $\beta_1^* \neq 0$, independently of the form of the return-flow function on the curve of the reestablishment of the pressure, there should be two parallel lines, separated by a distance of $-\ln\sqrt{\omega}$.

2. Let us consider the problem of the reestablishment of the pressure in a borehole, when the blocks have the form of spheres. Here, it is also necessary to solve the equation of piezoconductivity

$$\frac{\partial^2 u_2}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial u_2}{\partial \rho} - \frac{1}{\varkappa}\frac{\partial u_2}{\partial t} = 0 \tag{2.1}$$

under the following initial and boundary conditions:

$$\begin{gathered} u_2(\rho, r, t)|_{t=0} = 0 \\ u_2(\rho, r, t)|_{\rho=R} = u_1(r, t), \quad \left.\frac{\partial u_2(\rho, r, t)}{\partial \rho}\right|_{\rho=0} = 0 \end{gathered} \tag{2.2}$$

where ρ is the instantaneous coordinate; R is the radius of a sphere, i.e., the characteristic linear dimension.

Carrying out transformations, we obtain

$$U_2^\circ = U_1 3\left(\frac{\operatorname{cth}\delta}{\delta} - \frac{1}{\delta}\right) = U_1\Phi(s)$$

Further, analogously to the preceding case, we have

$$U_1(r_b, s) = \frac{q_0\mu}{2\pi k_1 h}\frac{1}{s}\left(-\ln\frac{\gamma r_b}{2}\sqrt{\frac{s}{\varkappa}\Phi(s)}\right) \tag{2.3}$$

Transforming (2.3), we obtain

$$\begin{aligned} U_1(r_b, s) = \frac{q_0\mu}{4\pi k_1 h}\Big[\frac{1}{s}\ln\frac{4\varkappa R}{3\gamma^2\sqrt{\varkappa_2 r_b^2}} - \frac{1}{2}\frac{\ln s}{s} \\ + \frac{1}{s}\ln \operatorname{th}\delta - \frac{1}{s}\ln\left(1 - \frac{\operatorname{th}\delta}{\delta}\right)\Big] \end{aligned} \tag{2.4}$$

For large values of δ, it can be assumed that $\operatorname{th}\delta \approx 1$. Then, expanding the last term in a series, we obtain

$$U_1(r_b, s) = \frac{q_0\mu}{4\pi k_1 h}\left[\frac{1}{s}\ln\frac{4\varkappa R}{3\gamma^2\sqrt{\varkappa_2 r_b^2}} - \frac{1}{2}\frac{\ln s}{s} + \frac{1}{s}\sum_{n=1}^{\infty}\frac{1}{n\delta^n}\right] \tag{2.5}$$

Going over in (2.5) to the inverse transform, for small values of F_0, we obtain

$$\begin{aligned} P_b \approx \frac{1}{2}\Big[\ln\frac{4\varkappa t}{\gamma r_0^2} - \ln\sqrt{\gamma F_0} - \ln 3 + \frac{1}{\sqrt{\pi}}\sum_{n=1}^{\infty}\frac{2^n}{(2n-1)(2n-1)!!} \\ \times F_0^{(2n-1)/2} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}F_0^n\Big] \end{aligned}$$

or, limiting ourselves to the first terms in the series

$$P_b \approx \frac{1}{2}\left[\ln\frac{4\varkappa t}{\gamma r_b^2} - \ln\sqrt{\gamma F_0} - \ln 3 + \frac{2}{\sqrt{\pi}}\sqrt{F_0} + \frac{1}{2}F_0 + \frac{4}{9\sqrt{\pi}}\sqrt{F_0^3}\right] \tag{2.6}$$

It follows from the latter expression that, in the case under consideration, the slope of segment 1 is greater than the slope of the same segment for a stratum with a return-flow function corresponding to blocks in the form of unbounded plates. A curve of dependence (2.6), plotted from the same starting data as in the preceding problem, is shown in Fig. 2 (line DE).

For small values of δ, it is difficult to obtain the inverse transform of (2.3). Therefore, we represent $\Phi(s)$ in the form of the series [8]

$$\Phi(s) = \sum_{n=1}^{\infty} \frac{6}{\delta + n^2\pi^2} \tag{2.7}$$

or

$$\Phi(s) = \sum_{n=1}^{\infty} \frac{6}{\delta + n^2\pi^2} + 1 - \sum_{n=1}^{\infty} \frac{6}{n^2\pi^2} \tag{2.8}$$

since, further on,* this expression for Φ (s) will be used for small values of n. The latter expression can be obtained from the relationship [9]

$$\frac{1}{s}\Phi(s) = 1 - \sum_{n=1}^{\infty} \frac{6}{n^2\pi^2} \exp(-n^2\pi^2 F_0)$$

using the rule of operational computation $f'(t) = sF(s) - f(0)$.

Thus, (2.3) is written as follows:

$$U_1(r_b, s) = \frac{q_0\mu}{2\pi k_1 h} \frac{1}{s}\left[-\ln\frac{\gamma r_b}{2}\sqrt{\frac{s}{\varkappa}\left(\sum_{n=1}^{\infty}\frac{6}{\delta + n^2\pi^2} + 1 - \sum_{n=1}^{\infty}\frac{6}{n^2\pi^2}\right)}\right] \tag{2.9}$$

Limiting ourselves in (2.9) to the number of terms in the sums: n = 1, n = 2, n = 3, n = 4, n = 5, etc., and finding consecutively the inverse transforms $U_1(r_b, s)$ we can plot the curve for reestablishment of the pressure for small values of δ (Fig. 2).

Thus, at n = 1

$$P_b \approx \frac{1}{2}\left[\ln\frac{4\varkappa t}{\gamma r_b^2} - E_i(-\pi^2 F_0) + E_i(-2.5506\pi^2 F_0)\right]$$

for n = 5

$$\begin{aligned} P_b \approx \frac{1}{2}\Big[\ln\frac{4\varkappa t}{\gamma r_b^2} &- E_i(-\pi^2 F_0) - E_i(-4\pi^2 F_0) - E_i(-9\pi^2 F_0) \\ &- E_i(-16\pi^2 F_0) - E_i(-25\pi^2 F_0) + E_i(-2.0492\pi^2 F_0) \\ &+ E_i(-6.0805\pi^2 F_0) + E_i(-12.2058\pi^2 F_0) \\ &+ E_i(-20.6557\pi^2 F_0) + E_i(-41.5842\pi^2 F_0)\Big] \end{aligned}$$

The calculations show that, already at n = 1, the curve obtained using this method has a common section with the curve described by formula (2.6). With an increase in the value of n, the region of coincidence of these curves increases (Fig. 2). This means that a curve for the reestablishment of the pressure can be plotted also for small values of F_0 (large values of δ); however, in this case, great difficulties arise in the calculations.

From the curve for the reestablishment of the pressure (curve DE on Fig. 2) it is evident that, in the case under consideration, the initial segment is located below the same segment for a stratum with a return-flow function corresponding to blocks, i.e., to unbounded plates. This is explained by the high rate of the return flow into the blocks, with a return-flow function corresponding to a spherical form of the block, which permits a smaller amount of liquid to arrive at the borehole through the cracks.

The difference in the ordinates of the initial segments increases in the direction of a decrease in the shutdown time, and approaches ln 3 at small values of $\varkappa_2/R^2$. In other words, the slopes of the initial segments for small values of $\varkappa_2/R^2$, in spite of the different return-flow functions, are practically identical at the start of reestablishment of the pressure, and only with an increase in the shutdown time does the slope

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*With $n \to \infty \left(1 - \sum_{n=1}^{n} \frac{6}{n^2\pi^2}\right) \to 0.$

of the segment corresponding to spherical blocks start to rise; this is explained by the radial character of the filtration of the liquid in spherical blocks.

If a semi-infinite body is taken as a model of the block, the curve of the reestablishment of the pressure in the borehole will have the same slope as the initial segment of the curve for the reestablishment of the pressure for a stratum with blocks in the form of infinite plates.† Consequently, the return flow of the liquid into the blocks, i.e., the plates, at first takes place in the same way as into semi-infinite bodies. This type of return flow is observed until the bounded character of the plate with respect to its thickness starts to have an effect. This corresponds exactly to the end of the initial segment 1 on the curve for the reestablishment of the pressure (Fig. 2).

From superposed curves for the reestablishment of the pressure, and from the formulas describing the curves, it follows that at $\beta_1^* = 0$, the slopes of the initial segments at the start of the reestablishment of the pressure for strata with blocks of any given form, of sufficiently large sizes, not necessarily of identical form and dimensions, will be exactly the same, equal to half of the slope of the asymptotic curve for the reestablishment of the pressure with a large value of t, and will correspond to blocks acting as semi-bounded bodies.

Attention must be called to the fact that the slope of the initial segment for a stratum with a return-flow function corresponding to spherical blocks is greater over its whole extension than the slope of the initial segment for a stratum with a return-flow function corresponding to blocks, i.e., to infinite plates.

For a stratum with real blocks, since they are of different sizes, this special characteristic should appear even more sharply. Therefore, the initial segment (and the transitional segment at $\beta_1^* \neq 0$) of the curve for the reestablishment of the pressure cannot be horizontal, as follows from [1], and only at the limit can it attain half of the asymptotic slope, at large values of t (the dotted line in Fig. 1). A curve for the reestablishment of pressure with a horizontal segment has no physical meaning (the liquid flows to the borehole, but the pressure is not reestablished).

The initial segment for a stratum with a return-flow function corresponding to a geometrical body of another form, for example a cube (with equal characteristics of its linear dimensions) will lie between the initial segments for a plate and a sphere.

3. In the article by G. I. Barenblatt, Yu. P. Zheltov, and I. N. Kochin [10], it is shown that the following connection exists between the lag time of the redistributed pressure τ_3 and the parameter $\alpha \sim k_2\sigma^2$:

$$\tau_3 = \frac{\eta}{\varkappa} = \frac{k_1}{\alpha\varkappa} \sim \frac{1}{\sigma^2\varkappa_2}$$

Since the specific friction surface $\sigma^2 \sim 1/l^2$, then

$$\tau_3 \sim l^2 / \varkappa_2 \tag{3.1}$$

Relationship (3.1), obtained on the basis of the theory of dimensionality, does not permit determining the characteristic linear dimension with respect to a known value of τ_3. And what is more, there are no estimates indicating the possible limits of the dimensions of blocks corresponding to an actual value of τ_3. Nevertheless, such evaluations can be made, starting from the results obtained above. Thus, for a stratum with a return-flow function corresponding to blocks, i.e., to infinite plates, relationship (3.1) goes over immediately into the equality

$$\tau_3 = R^2 / \varkappa_2$$

For a stratum with a return-flow function corresponding to spherical blocks, such a value of τ_3, determined in accordance with [4], is equal to $R^2/9\varkappa_2$ (i.e., this same value of τ_3 will correspond either to half the thickness of the plate, or to ⅓ the radius of the sphere). In general form, this dependence may be represented thus:

$$\sqrt{\tau_3\varkappa_2} = Rv \tag{3.2}$$

† It is easily shown that this is actually so, using the same method of a set of equations in operational form.

where Rv is the generalized dimension of the body which, for an infinite plate is equal to R, for an infinite cylinder to $^1/_2$ R, and for a sphere to $^1/_3$R [6].

Thus, from the value of τ_3, only the generalized dimension of the block can be found.

Using the concept of the generalized dimension of a block, let us evaluate the possible error in determining the characteristic linear dimension of the block from a known value of τ_3.

Let us assume that the return-flow function corresponds to spherical blocks, while, in determination of the characteristic linear dimension of the blocks, the return-flow function was assumed to correspond to blocks, i.e., to infinite plates. Then, the value of l found will obviously be three times less than the actual value. And, on the contrary, we take the starting return-flow function corresponding to blocks, i.e., to infinite plates, the value of l found will be approximately three times greater than the actual value. In other words, the maximal errors in determination of the characteristic linear dimension of the blocks from the lag time of the reestablishment of the pressure can lie only within the above limits. Actually, in determination of the characteristic linear dimension of the blocks of a real stratum, with application, for example, to a stratum with blocks, i.e., infinite plates, the error will be less.

Let us now evaluate the possible time required for the appearance, on the curve for the reestablishment of the pressure, of the characteristic segment necessary for determining τ_3. To this end, we use data pertaining to the Karabulak–Achaluki Lower Cretaceous deposit. For the rock of the blocks of this deposit, the value of the permeability varies from $0.1 \cdot 10^{-12}$ to $0.001 \cdot 10^{-12}$ m^2; the porosity of the rocks in the blocks, on the average, is 13%; the viscosity of the petroleum is $0.26 \cdot 10^{-3}$ N-sec/m^2; the compressibility coefficient of the liquid is $25.2 \cdot 10^{-2}$ m^2/N. The piezoconductivity of the blocks, calculated roughly from these data, is equal to $\varkappa_2 = 1.2\text{-}1.2 \cdot 10^{-2}$ m^2/sec.

We assume that the stratum is made up of infinite porous plates. Then, with a mean value of the dimensions of the blocks equal to 0.3m (R = 0.15m), the lag time of the reestablishment of the pressure $\tau_3 = 2\text{-}2 \cdot 10^{-2}$ sec. Therefore, in practice, it is impossible to obtain a curve for the reestablishment of the pressure having an initial segment.

However, with a mean value of the size of the blocks equal to 6m (R = 3m), τ_3 = 7.5-750 sec.

Under these conditions, it already becomes possible to obtain a curve for the reestablishment of the pressure, having an initial segment.

It should be noted that an analogous result has been obtained by V. P. Stepanov, using a somewhat different method.

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